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Damage mechanics: accomplishments, trends and needs

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Abstract

The objective of this study is to highlight the accomplishments, weaknesses and trends of damage mechanics and research needed for further development. The growing interest in damage mechanics is a proof that the accomplishments are significant. However, one of the messages is that the damage mechanics, in its focus on the dilute density of micro-defects and homogeneous solids, did not address the problems that are of primary interest in applications. The list of references in this paper is restricted to the current papers that list the older works. © 1999 Elsevier Science Ltd. All rights reserved.

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1. Introduction

Damage mechanics (DM) is a relatively new field of inquiry into the response and reliability of materials weakened by many randomly distributed microcracks of irregular shapes and random in size and orientation. The initial model (Kachanov, 1958) postulated that the loss of stiffness and integrity attributed to microcracks can be measured by a deterministic, macroscopic damage parameter. A corollary to this postulate is that the change of macroscopic response (effective parameters), engendered by a stochastic evolution of damage is both deterministic and gradual. More specifically, that the history of inelastic deformation and its change may be defined by the evolution of an internal variable that depends on the *expected value* of the micro-defect density.

DM models apply to the class of irreversible rearrangements of microstructure driven by a *gradual* evolution of micro-defects density observed as the gradual loss of chemical bonds on the atomic and reduction of the stiffness on macro scale. In principle, DM can be applied to porous materials weakened by micro-voids of all shapes. However, the microcracks, defined as cracks, commensurate to the

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characteristic material texture length that supports a local discontinuity of normal and tangential deformations, attracted, and still attract, most interest due to its relevance to the structural reliability and failure. Thus, this study will focus on microcracks and their propensity for unstable growth.

Damage evolution includes nucleation of new, and propagation and clustering of existing microcracks. Driven by the stress concentration (hot spots) microcracks grow along the internal surfaces of poor cohesive strength (weak links such as grain boundaries, interfaces separating phases of different properties) and heterogeneities (inclusions) which are also typical microcrack nucleation sites. In general, both hot spots and weak links are randomly distributed within the material microstructure.

2. State of art and accomplishments

2.1. Continuum models

The basic question of whether a formal structure of damage models exists was only recently explored by proposing two formal structures in addition to the traditional formalism of internal variables. The first of these formalisms (De Giorgi, 1993), in which the displacement field and discontinuity set are two unknowns, is based on the variational description of free discontinuities. Within the multifield description to each 'material particle' (Augusti and Mariano, 1996) is attached a finite-dimensional differentiable manifold to provide information of material texture and locations within Euclidian space. Mariano and Augusti (1997) discussed the axiomatic framework of damage mechanics, including metamodel of damage, meaning of damage potential, relation between DM and plasticity and the thermodynamic processes that are far from equilibrium.

The traditional local continuum models of non-equilibrium, irreversible deformation processes are based on the thermodynamics with internal variables and principle of determinism, material objectivity and local action. Principle of determinism implies that future depends on the past. The principle of local action implies that the effect of micro-defects, which are randomly distributed within an 'infinitesimally small neighborhood' of a material point **x**, on the macroscopic material parameters and fields can be described by an internal (damage) variable(s) $\mathbf{D}(\mathbf{x},t)$ that depends on the density and orientation of micro-defects but not on their location. The corollary is that the effective properties of the volume do not dependent on its size if the material is *statistically homogeneous*. The smallest volume for which this is true is the *representative volume element* (RVE) (Ostoja-Starzewski, 1993; Nemat-Nasser and Hori, 1993; Krajcinovic, 1996 etc.). Despite the opinion to the contrary a material can be statistically homogeneous even when the defects interact.

The simplest class of local continuum models (Lemaitre, 1992 and Krajcinovic, 1996), based on the thermodynamics of internal variables, implicitly assumes that each non-equilibrium state is *close* to its accompanying state that is equilibrated by a set of fictitious thermodynamic forces conjugate to the set of internal variables. The dependence on history is replaced by the dependence on what it produced (Rice, 1975). Assuming that the strains are infinitesimal and thermodynamics with internal variables applicable the relation between the macroscopic strain $\bar{\varepsilon}$ and stresses $\bar{\sigma}$ is

$$\bar{\varepsilon} = \bar{S}(D):\bar{\sigma} + \bar{\varepsilon}^{p} \tag{1}$$

In Eq. (1) $\bar{S}(D)$ is the effective compliance tensor, $\bar{\varepsilon}^{p}$ plastic strain, and bars above symbols stand for averaging over a RVE. Assuming that the rates of all fields (identified by a dot above symbols) are continuous, and the thermodynamic state stable, the rate form of Eq. (1) is

$$\dot{\bar{\varepsilon}} = \bar{S}:\dot{\bar{\sigma}} + \dot{\bar{S}}:\bar{\sigma} + \dot{\bar{\varepsilon}}^{p} = \dot{\bar{\varepsilon}}^{e} + \dot{\bar{\varepsilon}}^{d} + \dot{\bar{\varepsilon}}^{p} \quad \text{where } \dot{\bar{\varepsilon}}^{d} = \dot{\bar{S}}(D,\dot{D}):\bar{\sigma} \tag{2}$$

Superscripts e, d and p stand for the elastic, damage and plastic deformations. Assuming that a scalar damage potential Ω exists the rates of effective compliance and 'damage' (associative) strain are defined by the normality rule (Lubarda et al., 1994) as

$$\dot{\bar{S}}_{ijmn} = \dot{\lambda}\frac{\partial\Omega(\Gamma,\bar{S})}{\partial\Gamma_{ijmn}} \quad \text{and} \quad \dot{\bar{\varepsilon}}^{d}_{ij} = \dot{\lambda}\frac{\partial\Omega(\sigma,\bar{\varepsilon}^{d})}{\partial\bar{\sigma}_{ij}} \tag{3}$$

In Eq. (3) $2\Gamma_{ijmn} = \bar{\sigma}_{ij}\bar{\sigma}_{mn}$ is the thermodynamic force conjugate to the change of effective compliance and $\dot{\lambda}$ is the rate of a monotonically increasing scalar measure of cumulative damage λ. Assuming that the initial state $(\bar{S},\bar{\sigma},\bar{\varepsilon},\bar{\varepsilon}^{p})$ known and potential Ω given all other states can be determined from expressions (1) to Eq. (3). As in plasticity, the initial state is considered to be pristine ($D = 0$). Hence, the analytical solution is limited to the estimates of damage accrued during a particular loading path.

From the second law of thermodynamics, the irreversible entropy production rate $\dot{\eta}$ is

$$T\dot{\eta} = \dot{\bar{S}}_{ijnm}\Gamma_{ijmn} - \dot{U}^{s} \geq 0 \tag{4}$$

where T and U^s are the absolute temperature and energy converted into forming new surfaces within the volume.

An idiosyncrasy of damage is that only *active* (or open) microcracks, that support local discontinuity of the deformation field, affect the macro-response. The *passive* (or closed) microcracks may, however, become active during a non-proportional loading. Thus, the macro stress and strain tensors in (Eq. (1), Eq. (2)) must be decomposed into positive and negative projections (Ortiz, 1985; Yazdani and Schreyer, 1990; Lubarda and Krajcinovic, 1995; Krajcinovic, 1996) to account for the microcrack status. Both active and passive parts of the damage parameters must be recorded to predict the discontinuities of stress and strain gradients along the loading paths during which the sign of macro-stresses changes.

Leaving the damage modeling in composites to another chapter some of accomplishments will be listed below. Uniaxial compression models of Ortiz (1985) and Lubarda and Krajcinovic (1995) provide a formal insight into a more complex model for the consideration of a general case. However, the dominant cause of inelastic compressive deformation is the frictional resistance of sliding microcracks and their closure in unloading. Thus, a rational continuum model of a specimen in compression must be non-local and non-associative to capture the effects of crack interaction and frictional resistance on the crack closure. The frictional resistance to slip of two mating surfaces of a crack in unloading may be often misinterpreted as ductile deformation.

The problem of plastic-damage interaction and formulation of the rate form of a rational damage-elasto-plasticity theory was formally addressed by Chaboche (1988); Chaboche (1993); Lubarda and Krajcinovic (1995), Voyiadjis and Park (1996) and others. However, a formalism is useful only when the interaction of ductile and brittle deformation modes in a material is resolved by the physics on the microscopic scale. It is unlikely that guesses, inferences and phenomenological assumptions will help to solve this problem.

Non-local continuum damage models were suggested by Bazant and Pijaudier-Cabot (1988) and Bazant and Cedolin (1991). Allen (1994) and Herakovich (1998) studied evolution of damage in laminates, fatigue was studied by Talreja (1987), Lemaitre and Plumtree (1979), Lemaitre (1992) and Dvorak et al. (1994), while Ju (1989) and Ladeveze (1995) studied the computational aspects of damage.

2.2. *Micromechanics of damage*

The arbitrary selection of damage parameter and constitutive relations is a burden encumbered during the early phases of development of DM. Some of these arbitrary concepts and continuum models were later repudiated by micromechanical models that is, in combination with linear elastic fracture mechanics (LEFM), rightfully credited for the development of the scientific basis of DM.

To be acceptable an internal variable must be identifiable, measurable and related to the dominant mode(s) of irreversible rearrangements of the material microstructure. To this end the damage flux at a 'material point' $\boldsymbol{x}$ must be related to the microcracking within the corresponding RVE in the same sense that the plastic strain at $\boldsymbol{x}$ is related to the slip on the slip planes within RVE. The damage at $\boldsymbol{x}$ is defined by a configuration space that consist of a set of all possible values of microcrack sizes $a^{(k)}$, shapes, locations $\vec{x}^{(k)}$ and orientations $\vec{e}^{(k)}(\theta,\phi)$ for $0 < k \leq N$, where N is the number of defects per volume. The crack sizes, shapes, locations and orientations are random variables. The complete statistical description of the microcracks is provided by the joint distribution functional combined into the probability functionals as $N \rightarrow \infty$. Assuming that the statistics of micro-defects is known their effect on the effective material properties can, at least in principle, be determined using the effective field or medium micromechanical models. Hence, the damage evolution can be measured indirectly by its effect on the change of effective (macro) properties. The derivation of effective properties (Nemat-Nasser and Hori, 1993; Krajcinovic, 1996) is typically confined to dilute concentrations of micro-defects randomly distributed within a volume of a homogeneous, isotropic and elastic matrix. DM models based on the probability functional of microcrack would be too complicated to be useful even if the complete statistical description of damage were measurable in each 'material point' during the entire deformation process.

Assuming that the volume is not smaller than RVE each joint probability distribution depends on the distance between defects $|\vec{x}^{(k)} - \vec{x}^{(j)}|$ but not on their precise locations $(\vec{x}^{(k)},\vec{x}^{(j)})$. Damage characterization is further simplified by the assumption that the microcrack shapes are irrelevant and that the sizes and orientations are not correlated. The corresponding density function of penny-shaped cracks of radius a

$$w(a,\theta,\phi) \approx \vartheta(a)\rho(\theta,\phi) \tag{5}$$

is often depicted as a *rosette* histogram. As the consequence of linearity of linear elastic fracture mechanics the volume effective compliance $\bar{S}_{ijmn}$ admits the additive form

$$\bar{S}_{ijmn} = S^0_{ijmn} + S^*_{ijmn} \tag{6}$$

where S^0 is compliance at pristine state. The enhancement of compliance attributed to penny-shaped microcracks is then

$$S^*_{ijmn} = \int_{a^-}^{a^+} a^3\rho(a)da \int_0^{2\pi}\int_{-\pi/2}^{\pi/2} F_{ijmn}(\bar{S},\theta,\phi)\cos\phi d\theta d\phi = \omega\Phi_{ijmn}(\bar{S}) \tag{7}$$

(Krajcinovic, 1996). In Eq. (7) $\omega = N\langle a^3\rangle$ is Budiansky and O'Connell (1976) damage parameter for isotropic distribution of penny-shaped cracks. The normalized compliance due to a single crack $F_{ijmn}(\bar{S},\theta^{(k)},\phi^{(k)})$ is, in fact, Green's function for determination of Φ_{ijmn}. Described self-consistent micromechanical model approximates the compliance due to many inclusions within the pristine matrix by the superposition of compliances due to many single inclusions within the effective matrix. Details of the expression (7) change from one mean field model to the other. The analytical solution of Eq. (7) for $\bar{S}_{ijmn}$ is possible only for simple distribution of cracks since the Eq. (6) is implicit (i.e. $S^*_{ijmn} = S^*_{ijmn}(\bar{S}_{ijmn}), \Phi = \Phi(\bar{S})$) and the damaged matrix almost never isotropic. Finally, the mean field value of effective compliance can be determined even when the assumption Eq. (5) is not applicable.

The described algorithm was extensively employed in DM. Kachanov et al. (1994), Jasiuk et al. (1994) and Zheng and Collins (1997) considered defects of different shape. Horii and Nemat-Nasser (1985), Sumarac and Krajcinovic (1987), Zheng and Huang (1996) considered the defect imparted anisotropy. Kachanov (1993) and Ju and Tseng (1995) studied the effect of crack interaction on damage and effective properties. Krajcinovic (1997) derived the effective compliances at the percolation limit of defect densities. Fu et al. (1998) considered a novel computational algorithm for solution of many interacting defect problems. Two or three scale finite elements grids, connected by inter-scale connection operators, were developed by Fish et al. (1996) to consider damage in composites.

The characterization of a population of random micro-defects within an RVE entails precise and painstaking experimental measurements of the sizes, shapes, locations and orientation of $N \rightarrow \infty$ micro-defects and determination of statistical expectations (averages, higher order moments, correlation functions) of all random functions. To provide an analytical characterization of the micro-defects it is necessary that they are of simple shape and that the approximation (5) is realistic. The distribution $\rho(\theta,\phi)$ of micro-defects orientations can be then expanded into an infinite series of even order tensors and approximated by a single even order fabric tensor (Kanatani, 1984; Krajcinovic and Mastilovic, 1995). The accuracy of tensor approximations of the orientation distribution function $\rho(\theta,\phi)$ in Eq. (5) is proportional to the tensor order and inversely proportional to the defect density (Krajcinovic, 1996). The fact that the functional dependence of effective compliance on the microcrack distribution is similar in the limit of the percolation damage density is reassuring (Krajcinovic, 1997).

The string of assumptions introduced to render the homogenization (Germain et al., 1983), defined as the mapping of the effective properties of RVE onto the corresponding 'material' point of a continuum, possible and tractable is not always representative of actual deformation. Hence, the expression (7) for the compliance is in many cases but a qualitative guide (scaling law for the flux) since the experimental determination of the precise statistics of micro-defects is not impossible.

An interesting modification of standard micromechanics models was, in the context of concrete and rock failure, suggested by Bazant et al. (1996). Within this scheme the micro-strains are computed on 28 or more microplanes of different orientations and the micro-stresses are then deduced from microplane constitutive laws inferred by fitting the test data. The model is generalized to moderately large strains and verified by comparison with the test data.

2.3. Kinetics of damage evolution

In principle, the tensors $\dot{\bar{S}}_{ijmn},\dot{\bar{\varepsilon}}^{d}_{ij}$ can be determined from damage potentials $\Omega(\Gamma,\bar{S}),\Omega(\sigma,\bar{\varepsilon}^{d})$ Eq. (3) by invoking the postulate of *local dependence* (Rice, 1975); namely, when the propagation of a microcrack is attributed solely to the mean field value of their conjugate thermodynamic forces. In most cases the damage potential is inferred from the test data (Ashby and Sammis, 1990). The potential $\Omega(\Gamma,\bar{S})$ is more useful since the damage flux since $\dot{\bar{\varepsilon}}^{d}$ can be derived knowing $\dot{\bar{S}}_{ijmn}$ from (2.b) while the opposite is not true. Since the statistics of microcracks is not known it seems that the only identifiable and measurable continuum damage parameter is $\bar{S}_{ijmn}$ (Krajcinovic, 1996). The rate of the effective stiffness does not measure the damage evolution itself but the effect that the microcracking has on the macro response in the same sense that the rate of plastic strain measures the effect of the slips along the slip planes has on the inelastic strains. Moreover, the described derivation of damage potentials from micro-potentials is not correct unless it takes in consideration the crack interaction (including potential change of state and closure) and, most importantly, the effect of energy barriers on the propagation (or trapping) of microcracks.

Kinetics of damage evolution can also be deduced from lattice simulations (Krajcinovic, 1996; van Mier, 1997). Some of the recent results (Krajcinovic and Vujosevic, 1998; Mastilovic and Krajcinovic, 1999) focused on the determination of universal trends (Hansen et al., 1991) of response may, indeed, prove to be very useful.

2.4. Experimental characterization of damage

Experimental characterization of microcrack populations and its evolution is, due to their size, a daunting task. Advanced methods of quantitative non-destructive characterization of internal damage are still in development stage (Achenbach, 1992). The basic statistics of accrued damage is most often determined by stereological methods (Budiansky and O'Connell, 1976) from the observations of sections of a damaged specimen (Nemati et al., 1998). Acoustic emission test (Holcomb and Costin, 1986; Lockner et al., 1991) is used to deduce the microcrack evolution patterns in a qualitative sense. The large number of data renders the computerized digital image processing and analyses (Mobasher et al., 1990) of damage absolutely necessary.

The experimental challenges are many. The identification of passive (closed) cracks obscured by the texture features of similar size, and the dependence of data on the magnification and image resolution are vexing and often mentioned problems. Additionally, most of the test data are recorded on materials with large features (concrete, rocks) in a homogeneous state of strain. The number of very delicate tests needed to form a representative sample required for the identification of universal trends will discourage even the most persistent experimentalist. It seems that in the absence of detailed micro-scale data the damage characterization of damage from measurements of effective compliance (Audoin and Baste, 1994) is currently the only realistic strategy. Unfortunately, this characterization is useful only within the mean-field regime (dilute damage) of deformation. Furthermore, while a unique effective compliance corresponds to a given distribution of microcracks the inverse is not true.

3. Research needed

Despite its many accomplishments many essential aspects of DM are in dire need of further research. Analytical modeling of the growth of short (micro) cracks within a material characterized by a random distribution of barriers (capable of trapping cracks) is a serious task that is more than matched by the complexity of experimental observations. Distributions of local stresses (that drive the damage) and material texture (that resist damage evolution) are statistical in nature. Moreover, this statistics changes as the damage evolves. The damage evolution can be stable and gradual in ductile and *damage tolerant* (micro-heterogeneous) materials that are characterized by a wide bandwidth of barrier strengths. In the case of *damage sensitive* (micro-homogeneous) and ductile materials, with a narrow band of barrier distribution, the microcrack growth can be stable only in the absence of long range tensile stresses. Except for mean field response (dilute damage) the governing statistics is not that of averages but one of extremes (Bolotin, 1989). In both cases the microcrack propagation in an essential way depends on the statistics of the microstructure which is almost never considered.

Most current continuum and micromechanical models are focused on the uniaxial tension of homogeneous materials to take advantage of the deterministic micro structure and the fact that all microcracks are active and normal to the applied tensile tractions. However, the damage evolution in homogeneous (damage sensitive) materials subjected to long range tensile stresses is unlikely to be stable. The specimen failure will occur at very modest damage as soon as one of the microcracks becomes critical rendering the rest of damage irrelevant. Literature focusing on much more interesting problem of damage in brittle solids subjected to compressive tractions is meager. The micromechanically based non-associative model of a specimen in compression, that includes the friction over the mating surface of sliding microcracks as an additional energy dissipation mechanism (Nemat-Nasser and Obata, 1988 and Basista and Gross, 1998) is an important but a first step. The next step is to introduce the distribution of cohesive strengths within the material. Micromechanical models dealing with damage evolution in damage tolerant (composite) materials (Dvorak et al., 1994) are reviewed elsewhere in this issue.

To illustrate the current limitations and future needs of DM it suffices to discuss the deformation of a grained rock subjected to a slowly incremented axial contraction $\bar{\varepsilon}^a$ superimposed on a stationary hydrostatic strain $\bar{\varepsilon}^o$. A typical deformation process of this rock specimen consists of several phases (Reches and Lockner, 1994). Initially, the deformation is, at very small $\bar{\varepsilon}^o > \bar{\varepsilon}^a$, *elastic*. At larger but still modest values of applied shear strain $(\bar{\varepsilon}^a - \bar{\varepsilon}^o)/2$ (*local process regime*) microcracks are nucleated at local stress concentrations and/or weak planes. The microcrack density is dilute and their growth is, in the absence of long-range tensile strains, stable. The material is statistically homogeneous on a rather small scale, damage evolves gradually and the local continuum and micromechanical models (based on Eq. (1) to Eq. (7)) are applicable. On the macro scale the evolving damage is manifested by gradually decreasing specimen stiffness.

As the applied shear stress is further enhanced the distance separating adjacent microcracks diminishes and the effect of the interaction of closely located cracks on their propagation increases. Within this *non-local process* regime damage is driven by the interaction enhanced growth of correlated cracks that form clusters (shear bands). The macro response remains stable as long as the length of specimen length L exceeds the size of the largest cluster (defined by the correlation range ξ). The increase of the damage evolution rate can be observed macroscopically as an increased rate at which the specimen stiffness diminishes. The material is still local on the specimen scale that explains the low scatter of test data. Expressions (1) to Eq. (7) are applicable only on the specimen size. As a consequence of the crack interaction the bandwidth of the stress distribution increases and shifts toward large values. The finite element modeling of this regime requires non-local constitutive equations (imbrication) and extensive re-meshing.

At large confinements $\bar{\varepsilon}^o \gg \bar{\varepsilon}^a$, at which all long-range principal stresses are compressive, the cracks nucleated at heterogeneities cannot grow. The damage density can increase to the percolation limit (at which the specimen is disconnected into two parts) and beyond. In the final limit the material is comminuted and the deformation approaches the granular flow. At percolation transition the material is self similar and the effective material properties and fault mass (volume) are defined by fractal scaling laws (Krajcinovic, 1996; Krajcinovic, 1997). The governing statistics is that of extremes and the stress distribution is multifractal (Hansen et al., 1991; Krajcinovic and Mallick, 1995).

At lesser confinements the local stresses at crack tips can be rendered tensile by interaction with its neighbors. As a cluster of interacting (correlated) cracks grows the elastic energy release stored at the cluster tip increases rendering the growth even faster. The synergetic effect of microcrack interaction on the rate of cluster elongation, known as *cooperative effect*, leads ultimately to rapid localization of strain in the largest cluster (fault) as $\xi \rightarrow L$. The specimen in the *transformed regime* consists of two phases. The damage density in and effective compliance of the material in the interior of the fault are much larger than those of the exterior. Hence, the specimen is not statistically uniform and none of the equations in this review applies. The response strongly depends on the shape and size of the specimen and details of test equipment. In the absence of the ergodicity the scatter of test data recorded on 'equal' specimens is large.

Modeling of the softening regime (defined by the negative specimen tangential stiffness) is an interesting and controversial problem. The material of the softening specimen cannot be statistically uniform without violating the second law of thermodynamics Eq. (4). According to the original solution (Rice, 1976), based on Hadamard's stability theory, the transition from hardening to softening regime is instantaneous and manifested by a cusp in the stress-strain curve. The fault also forms instantaneously across the specimen and does not change afterwards. Moreover, the thickness of the fault is equal to zero such that the loss of homogeneity is not an issue. The localization limiters (Belytschko et al., 1986), non-local theory (Bazant and Pijaudier-Cabot, 1988) and several other strategies were used to alleviate the pathology of fault growth and geometry. Finally, the results of particle dynamic simulations (Krajcinovic and Vujosevic, 1998) are in full agreement with be test data (Lockner et al., 1991)

4. Summary

The goal of this narrative is not only to highlight some of the accomplishments but also to reaffirm the practical value of DM. However, the application of DM models is currently limited to modeling deformation of ductile and damages tolerant materials (composites) regardless of the stress sign and to damage sensitive materials (rocks, concrete, silicone, epoxy, iron, bones, etc.) subjected to the macro-scale compression.

The fact that the DM is not frequently used in industry can be blamed on the early models focus on the wrong problem, i.e. on damage in damage sensitive specimens subjected to tensile tractions. Secondly, as argued in this study most DM models focus on the least eventful and important part of the deformation (local process regime) and dilute concentration of micro-defects. Finally, the experimental data are not always in tune with the analytical models with arbitrary and poorly identified measures of damage.

Thus, it is not surprising that the aircraft design concepts of reliability, damage tolerance, multiple damage sites and residual strength have little in common with DM as defines herein. The damage can, both rightly and wrongly, be perceived as an overture to failure. Indeed, it seems to this author that DM must address the reliability of damaged structures to convince the industry of its practical value. This task will require a more detailed (and different) statistical characterization of the thermodynamic state (distribution of hot spots and weak links) and its change as the damage evolves. To say this is one thing, to do it is still other. In view of the complexity of in-line experimental characterization of damage it seems that the practical solution, at least a temporary one, should be sought in computer simulations coupled with the methods of statistical mechanics (or computational micromechanics Ortiz, 1996).

A combination of DM and plasticity is applicable to metals in which a growing microcrack can be trapped by the ductile deformation (dislocation bands). Further progress in the application of DM models to the problems of fatigue, creep, comminution, high velocity impact on and penetration through solids, materials subjected to very low (cryogenic) temperatures, corrosion, aging, solidification, hydraulic and thermal fracturing, etc. is a matter of time.

5. Conclusions

The current weaknesses of DM provide the most fertile fields of future research. For example, experimental characterization of damage and its evolution during deformation are certainly both a weakness and an interesting field of work. Characterization of damage and reconstruction of damaged material from the test data (Yeong and Torquato, 1998) is another challenge waiting resolution.

From a purely analytical viewpoint the characterization of a thermodynamic state is a reasonably well investigated on all scales. The same cannot be stated for the change of a thermodynamic state. The damage potentials (and damage functions) are based on flimsy grounds. A model is often assumed to be associative without given it too much thought. The statistical homogeneity and isotropy of damage are assumed even when the test data and simple reasoning does not supports these assumptions. Some of these sins were committed when we did not know better and in a total absence of test data. However, perpetuation of these mistakes and formulation of new continuum theories that are not based on physics and materials may actually be even harmful.

Finally, unlike plasticity DM cannot live and die investigating mean field (local process) regime.

Microcracks have a strong tendency to grow and become macrocracks and the thermodynamic states can be unstable. Hence, it is not the damage potential that should be sought but the limit function (surface). The transition from statistically homogeneous state to the state dominated by a large cluster(s) is very important. The day when a DM model will provide a realistic estimate of the residual strength of a damaged (aged) structure or the fatigue limit it will come of age. This, of course, does not mean that all that was done in the past is wrong. To the contrary, it seems to this author that the acceptance of DM is growing very rapidly. There is no doubt here that most if not all of the current weaknesses of DM will be soon become history.

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